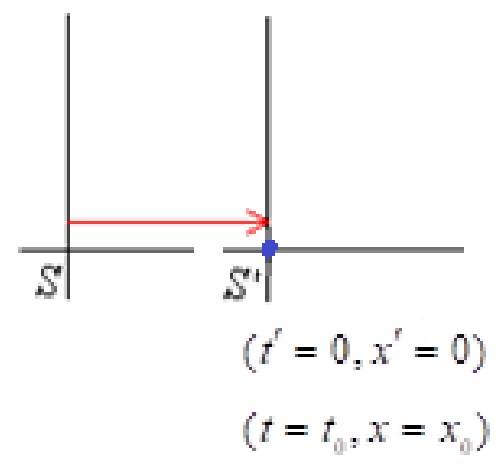
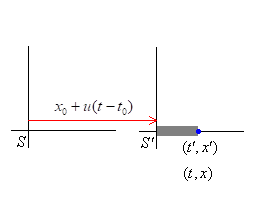
**Poincare´ Transformation**

Wanted to look at the Poincare´ transformation which allows for reference frames which don’t coincide at the origin at t = t´ = 0. Let S be a stationary frame, and S′ be a frame moving to the right with speed u. Let’s say t´ = 0, x´ = 0 when t = t0, x = x0. Now then, if an event happens in frame S at the coordinates (*t,x*), where/when does the event happen in S′, i.e., what are the coordinates (*t*′,*x*′)?

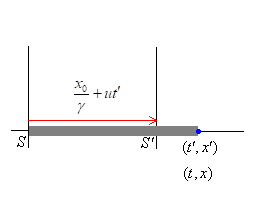
We’ll use the following construction. Consider a board (in grey) in the S′ reference frame with length x′. And suppose that a light is flashed at the end of the board at position (t′,x′) in S′. If you’re in S, you’d suppose that event to have ocurred at point (t, x). Furthermore, you say that:



since you’ll measure the length of that rod to be x΄/γ. Either way, we can solve for x΄ and get:



So that’s how the x′ coordinate transforms. What about the t′ coordinate? Well now we must consider the situation from the opposite perspective. Let’s say we have a board fixed in the S frame, and a light flashes off at coordinate (t,x). According to S´, by time t´ you’d be a distance x0/γ + ut´ away from the origin.



Further, you’d measure the coordinate of the event to have ocurred a distance x/γ from the origin. And so you’d say:



So now we want to solve for t΄ in terms of x and t. So equating our two expressions for x´, we’d have:



So our coordinate transformations are:



which we can also write as:



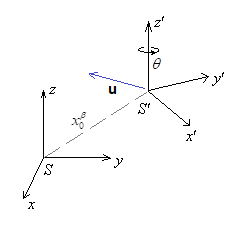
which makes sense. In matrix notation this would read:



And recall x0β is the coordinate of the origin of the primed reference frame when x´β = 0. Anyway, I think this is right. The Poincare transformation preserves the metric, just as the Lorentz transformation does. Perhaps the most general Poincare´ transformation would be something of the form,

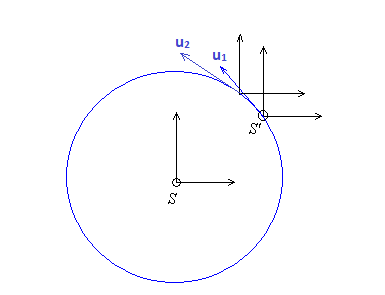


which describes both boost, rotation, and translation.



**Thomas precession**

Consider the special case of the reference frame S′ changing the direction of its velocity w/r to S. For example, S′ may be rotating around S, illustrated below, or it may be doing any general acceleration that involves a change in direction.



Now we’ll show that from the perspective of S´, it will appear to precess, while it rotates. Well actually we’ll just sketch the mathematical details. So consider the instantaneous reference frame of S´ when it is at coordinates 1 = (ct1, **x**1) with velocity **u**1, the coordinates of S´ are related to those of S via:



where we have explicitly indicated that S′ is not changing the orientation of its axes at all, since we have set **θ** = 0. And I used a subscript t1 to indicate that the primed coordinates are those with respect to this instantaneous reference frame. Now consider a short differential time later, t2 = t1 + δt, whereby the position of S´ has changed to 2 = 1 + δ = (ct1 + cδt1, **x**1 + δ**x**1) and the velocity has changed to **u**2 = **u**1 + δ**u**­. In this reference frame, the coordinates of S´are related to those of S by:



where again I have used the subscript t2 to remind us that the primed coordinates are with respect to *this* new reference frame. Now let us see how the coordinates t2 and t1 are related. So we’ll solve for in the first equation and substitute it into the second equation.



The first term is what we’re interested in. The last two terms just represent shifts of the origin of S´. So apropos the first guy, without loss of generality, we can let **β** be along the x-axis, and δ**β** can somewhere in the x-y plane – we can always orient our axes to make this so. In that case we have:



which is large and obnoxious. Still, expanding this out for small δβ = δβ1**i** + δβ2**j**, and only keeping first order terms we have:



To make further progress we’ll note that:



and so filling this into the matrices, multiplying them out, keeping only terms to first order in δ, and writing in terms of **S** and **K** we get:



where δ**β**∥ = δ**β**1 and δ**β**⊥ = δ**β**2. Now the **K** part of the expression is to be expected. It is just the matrix boosting the speed of the 1 rest frame from **β** to **β** + δ**β**. But the **S** part *is* unexpected. It is a pure rotation matrix, which rotates the ´t1 frame about an angle.



Now,



So we can simplify:



and write:



Now since this occurs in time δt, the rate of precession of S′ w/r to S is:



So we have:

****

Note that **ω**T is given by the simplest combination of the only two relevant vectors. We can see that maximum precession will occur when **a** and **u** are perpendicular to each other. This would happen for circular motion. Consider case of orbital rotation counter clockwise at rate Ω at radius R. Then the rate of precession would be:



which is in the opposite direction to the orbital motion. So could say that if an object S′ is rotating about S with angular momentum, **L**, then it will appear to S to have an angular momentum **L** given by:



(where IO is the moment of inertia about the origin). This seems to be the classical analogue to how spin naturally arises from the relativistic generalization of quantum mechanics. Actually, not so sure about combining the two angular momenta. Maybe should be careful about perspectives. Maybe S´ only appears to be rotating to itself, not to S? On the other hand, the velocity **u** and acceleration **a** and γ are clearly S-dependent quantities. So it does have something to do with S.